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Heterogeneous Beliefs in a Sticky-Price Foreign Exchange Model

Mikael Bask

University of Helsinki, RUESEG and HECER

and

Carina Selander

Umeå University

Discussion Paper No. 48

February 2005

ISSN 1795-0562

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Abstract

It is demonstrated in this paper that the exchange rate "overshoots the overshooting equilibrium" when chartists are introduced into a sticky-price monetary model due originally to Dornbusch (1976). Chartists are introduced since questionnaire surveys reveal that currency trade to a large extent is based on technical trading, where moving averages is the most commonly used technique. Moreover, the surveys also reveal that the importance of technical trading depends inversely on the planning horizon in currency trade. Implementing these observations theoretically, and deriving the exchange rate's perfect foresight path near long-run equilibrium, it is also demonstrated in this paper that the shorter the planning horizon is, the larger the magnitude of exchange rate overshooting. Finally, the effects on the exchange rate's time path of changes in the model's structural parameters are derived.

JEL Classification: F31, F41

Keywords: Exchange rates, moving averages, overshooting, planning horizon, technical analysis.

Mikael Bask

Department of Economics
P.O. Box 17 (Arkadiankatu 7)
University of Helsinki
FIN-00014 University of Helsinki
FINLAND

e-mail: mikael.bask@helsinki.fi

Carina Selander

Department of Economics
Umeå University
SE-901 87 Umeå
SWEDEN

e-mail: carina.selander@econ.umu.se

1 Introduction

Should foreign exchange models focus on observed behavior of exchange rates, or should the focus be on observed behavior of currency traders? Since a main purpose of economic theory is to develop models that can explain observed regularities, there is an obvious advantage of the first point of departure. Nevertheless, in order to develop an *economic* theory of exchange rate movements, one cannot disregard the behavior of those who actually trade in the foreign exchange market. Modeling observed behavior of foreign exchange traders is, however, not sufficient in order to obtain an economic theory since one must also explain *why* traders act as they do. The level of ambition in the present paper is to take a first step towards developing an economic theory of exchange rate movements by taking into account observed behavior of currency traders.

In November 1988, Taylor and Allen (1992) conducted a questionnaire survey for the Bank of England on the foreign exchange market in London. The survey covered 353 banks and financial institutions, with a response rate of over 60 per cent, and was the first to ask specifically about the use of *technical analysis*¹, or *chartism*, among currency traders. The results of the survey were striking, with two per cent of the respondents reported never to use fundamental analysis in forming their exchange rate expectations, while 90 per cent reported placing some weight on technical analysis at the intraday to one week horizon. At longer *planning horizons*, however, Taylor and Allen (1992) found that the importance of technical analysis became less pronounced.

That technical analysis is extensively used in currency trade has also been confirmed by Menkhoff (1997), who conducted a survey in August 1992 on the German market, by Lui and Mole (1998), who conducted a survey in February 1995 on the Hong Kong market, by Oberlechner (2001), who conducted a survey in the spring 1996 on the markets in Frankfurt, London, Vienna and Zurich, and, as a final example, by Cheung and Chinn (2001), who conducted a survey between October 1996 and November 1997 on the U.S. market. A general observation in these surveys is that a skew towards reliance on technical, as opposed to fundamental, analysis at shorter planning horizons was found, which became gradually reversed as the length of the planning horizon considered was increased.

Frankel and Froot (1986) were the first to use a *chartist-fundamentalist* setup in a foreign exchange model, where the heterogeneous behavior of currency traders was taken into account. In their model, a bubble in the exchange rate takes off and collapses since the portfolio managers² learn more slowly about the model than they are changing it by revising the weights given to the chartists' and fundamentalists' exchange rate expectations. A chartist-fundamentalist model is also developed by De Grauwe and Dewachter (1993), where the weights given to the chartists' and fundamentalists' expectations depend on the deviation of the exchange rate from its fundamental value. Specifically, more (less) weight is given

¹ For a theoretical description of technical trading techniques used in the foreign exchange market, the reader can turn to Neely (1997).

² Without affecting the theoretical results in this paper, we assume that it is the chartists and fundamentalists, and not the portfolio managers, who trade in currencies. Therefore, we leave the portfolio managers out of account in the model.

to the chartists' expectations when the exchange rate is close to (far away from) its fundamental value. In Levin (1997), however, as a final example, the relative importance of technical versus fundamental analysis does not change over time.

The specific purpose of this paper is to implement theoretically, the aforementioned observation that the relative importance of technical versus fundamental analysis in the foreign exchange market depends on the planning horizon in currency trade. For shorter planning horizons, more weight is placed on technical analysis, while more weight is placed on fundamental analysis for longer planning horizons. In the model developed, technical analysis is based on moving averages since it is the most commonly used technique among currency traders using chartism (e.g., Taylor and Allen, 1992, and Lui and Mole, 1998). Further, fundamental analysis is based on a sticky-price monetary foreign exchange model due originally to Dornbusch (1976).

The main questions in focus are: how is the dynamics of the exchange rate affected when technical analysis is introduced into a sticky-price monetary model? Specifically, will the exchange rate “overshoot the overshooting equilibrium”? This phrase was coined by Frankel and Froot (1990) when discussing possible explanations to the dramatic appreciation of the U.S. dollar in the mid-1980's. The “overshooting equilibrium” refers, of course, to Dornbusch (1976). Further, how is the planning horizon and the overshooting effect affected when market expectations³ are characterized by perfect foresight?

The remainder of this paper is organized as follows. The benchmark model and the expectations formations are presented in Section 2. The formal analysis of the model is carried out in Section 3, and Section 4 contains a short concluding discussion of the main results in this paper.

2 Theoretical framework

The benchmark model is presented in Section 2.1, and the expectations formations are formulated and discussed in Section 2.2.

2.1 Benchmark model

Basically, the model is a two-country model with a money market equilibrium condition, an international asset market equilibrium condition, a price adjustment mechanism since goods prices are assumed to be sticky, and market expectations that are formed by the relative weights given to the chartists' and fundamentalists' exchange rate expectations. The formal structure of the model is presented below, where Greek letters denote positive structural parameters.

The money market is in equilibrium when

$$m[t] - p[t] = \bar{y} - \alpha i[t], \quad (1)$$

³ Market expectations are the weighted average of the chartists' and fundamentalists' expectations, i.e., in a model with portfolio managers, like in Frankel and Froot (1986), market expectations coincide with the portfolio managers' expectations. See also footnote 2.

where m , p , \bar{y} and i are (the logarithm of) the relative money supply⁴, (the logarithm of) the relative price level, (the logarithm of) the relative real income, and the relative nominal interest rate, respectively. Moreover, m and \bar{y} are exogenously given⁵. Thus, according to (1), real money demand depends positively on real domestic income and negatively on nominal domestic interest rate. The money market is assumed to be permanently in equilibrium, i.e., disturbances are immediately intercepted by a perfectly flexible domestic interest rate.

The international asset market is in equilibrium when

$$i[t] = s^e[t+1] - s[t], \quad (2)$$

where s is (the logarithm of) the spot exchange rate, which is defined as the domestic price of the foreign currency. Moreover, the superscript e denotes expectations. The equilibrium condition in (2), also known as uncovered interest rate parity, is based on the assumption that domestic and foreign assets are perfect substitutes, which only can be the case if there is perfect capital mobility. Since the latter is assumed, only the slightest difference in expected yields would draw the entire capital into the asset that offers the highest expected yield. Thus, the international asset market can only be in equilibrium if domestic and foreign assets offer the same expected yield. According to (2), a positive (negative) relative nominal interest rate means that the exchange rate is expected to depreciate (appreciate). The equilibrium condition is maintained by the assumption of a perfectly flexible exchange rate.

The price adjustment mechanism is

$$p[t+1] - p[t] = \beta(s[t] - p[t]), \quad (3)$$

where $0 \leq \beta \leq 1$ and $s - p$ is (the logarithm of) the real spot exchange rate. According to (3), goods prices are assumed to be sticky. Thus, goods prices respond to market disequilibria, but not fast enough to eliminate the disequilibria instantly. Two extremes are obtained by setting $\beta = 0$, which is the case of completely rigid goods prices, and by setting $\beta = 1$, which is the case of perfectly flexible goods prices.

2.2 Expectations formations

According to questionnaire surveys (see cited references in Section 1), the relative importance of technical versus fundamental analysis in the foreign exchange market depends on the planning horizon in currency trade. For shorter planning horizons, more weight is placed on technical analysis, while more weight is placed on fundamental analysis for longer planning horizons. In the present paper, we formulate this observation as

$$s^e[t+1] = \omega(\tau) s_f^e[t+1] + (1 - \omega(\tau)) s_c^e[t+1], \quad (4)$$

⁴ That is, the difference between the domestic and foreign money supplies. The other macroeconomic variables in the model are defined in a similar way.

⁵ In the simulations of the model in Section 3.5, m will follow a stochastic process.

where s^e , s_f^e and s_c^e denote market expectations and expectations formed by fundamental analysis and chartism, respectively. Moreover, $\omega(\tau)$ is a weight function that depends on the planning horizon, τ :

$$\omega(\tau) = 1 - \exp(-\tau). \quad (5)$$

Technical analysis, or chartism, utilizes past exchange rates in order to detect patterns that are extrapolated into the future. Focusing on past exchange rates is not considered as a shortcoming for currency traders using this technique since a primary assumption behind chartism is that all relevant information about future exchange rate movements is contained in past movements. Further, fundamental analysis is based on a model that consists of macroeconomic fundamentals only, which in the present paper is a sticky-price monetary foreign exchange model due originally to Dornbusch (1976) (see the benchmark model in Section 2.1).

The most commonly used technique among currency traders using chartism is the moving average model (e.g., Taylor and Allen, 1992, and Lui and Mole, 1998). According to this model, buying and selling signals are generated by two moving averages; a short-period moving average and a long-period moving average, where a buy (sell) signal is generated when the short-period moving average rises above (falls below) the long-period moving average. In its simplest form, the short-period moving average is the current exchange rate and the long-period moving average is an exponential moving average of past exchange rates.

Thus, when chartism is used, it is expected that the exchange rate will increase (decrease) when the current exchange rate is above (below) an exponential moving average of past exchange rates:

$$s_c^e[t+1] = s[t] + \gamma(s[t] - MA[t]), \quad (6)$$

where MA is an exponential moving average of past exchange rates, i.e., the long-period moving average. Moreover, the long-period moving average can be written as

$$MA[t] = (1 - \exp(-v)) \sum_{k=0}^{\infty} \exp(-kv) s[t-k], \quad (7)$$

where the weights given to current and past exchange rates sum up to 1:

$$(1 - \exp(-v)) \sum_{k=0}^{\infty} \exp(-kv) = (1 - \exp(-v)) \cdot \frac{1}{1 - \exp(-v)} = 1. \quad (8)$$

Finally, when fundamental analysis is used, it is expected that the exchange rate will adjust to its fundamental value according to a regressive adjustment scheme:

$$s_f^e[t+1] = s[t] + \delta(\bar{s} - s[t]), \quad (9)$$

where $0 \leq \delta \leq 1$ and \bar{s} is (the logarithm of) the spot exchange rate in long-run equilibrium, i.e., the exchange rate's fundamental value. Note that when $\delta = 1$, it is expected that the exchange rate will be in long-run equilibrium the next time period.

3 Formal analysis of the model

The long-run effects in the model are derived in Section 3.1. Thereafter, in Section 3.2, the exchange rate overshooting phenomenon is investigated. Specifically, we will examine whether the exchange rate “overshoot the overshooting equilibrium”, i.e., if the magnitude of exchange rate overshooting in the present model is larger than in the Dornbusch (1976) model? The adjustment path to long-run equilibrium, when market expectations are characterized by perfect foresight, is derived in Section 3.3, and the *perfect foresight planning horizon* and the *perfect foresight overshooting effect*⁶ are scrutinized in Section 3.4.

Since the long-period moving average in (7)-(8) is a function of all past exchange rates, the model is not easy to analyze formally. But by assuming that the economy has, for a long time, been in long-run equilibrium before a monetary disturbance occurs, the moving average in (7)-(8) is (approximately) equal to the long-run equilibrium exchange rate:

$$MA[t] \approx (1 - \exp(-v)) \sum_{k=0}^{\infty} \exp(-kv) \bar{s} = \bar{s} (1 - \exp(-v)) \sum_{k=0}^{\infty} \exp(-kv) = \bar{s}. \quad (10)$$

The assumption in (10) simplify the analysis considerably, but will be relaxed in Section 3.5, where a small simulation study is accomplished in order to illustrate the behavior of the model.

3.1 Long-run equilibrium

Since it is assumed in this section that the exchange rate is in long-run equilibrium,

$$s[t] = \bar{s}. \quad (11)$$

Substituting (10) (assuming equality in the equation) and (11) into the expectations formation in (6), the expectations formed by technical analysis become

$$s_c^e[t+1] = \bar{s} + \gamma(\bar{s} - \bar{s}) = \bar{s}, \quad (12)$$

i.e., it is expected that the exchange rate is in long-run equilibrium. Moreover, substituting (11) into the expectations formation in (9), the expectations formed by fundamental analysis become

$$s_f^e[t+1] = \bar{s} + \delta(\bar{s} - \bar{s}) = \bar{s}, \quad (13)$$

i.e., it is expected that the exchange rate is in long-run equilibrium. Then, substitute the expectations formations in (12)-(13) into market expectations in (4):

$$s^e[t+1] = \omega(\tau) \bar{s} + (1 - \omega(\tau)) \bar{s} = \bar{s}, \quad (14)$$

i.e., the market expects that the exchange rate is in long-run equilibrium. Recall that the results in (12)-(14) are based on the assumption that the economy has, for a long time, been in long-run equilibrium before a monetary disturbance occurs.

⁶ That is, the planning horizon and the overshooting effect when market expectations are characterized by perfect foresight.

(The logarithm of) the relative price level⁷ in long-run equilibrium, \bar{p} , can be solved for by using the equations that describe the money and the international asset markets in equilibrium, i.e., (1)-(2):

$$\bar{p} = m[t] - \bar{y}, \quad (15)$$

since, according to (11) and (14),

$$s[t] = s^e[t+1] = \bar{s}. \quad (16)$$

Thus, the quantity theory of money holds in the long-run since, according to (15),

$$\frac{d\bar{p}}{dm[t]} = 1. \quad (17)$$

Moreover, since (16) as well as

$$p[t] = p[t+1] = \bar{p}, \quad (18)$$

hold in long-run equilibrium, the price adjustment mechanism in (3) reduces to

$$\bar{s} = \bar{p}. \quad (19)$$

Thus, purchasing power parity holds in the long-run since, according to (19),

$$\frac{d\bar{s}}{d\bar{p}} = 1. \quad (20)$$

Finally, the quantity theory of money and purchasing power parity, i.e., (17) and (20), implies that

$$\frac{d\bar{s}}{dm[t]} = \frac{d\bar{s}}{d\bar{p}} \cdot \frac{d\bar{p}}{dm[t]} = 1, \quad (21)$$

which is the long-run effect on the exchange rate of a change in money supply.

It should be stressed that the quantity theory of money and purchasing power parity results are not dependent on the simplifying assumption that the economy has, for a long time, been in long-run equilibrium before a monetary disturbance occurs. But even if the quantity theory of money and purchasing power parity hold in the long-run, there are short-run deviations from these one-to-one relationships. This is demonstrated in the next section on exchange rate overshooting.

3.2 Exchange rate overshooting

Using (10) (assuming equality in the equation) in the expectations formation in (6), and substituting the resulting equation as well as the expectations formation in (9) into market expectations in (4), we have that

$$\begin{aligned} s^e[t+1] &= \omega(\tau)(s[t] + \delta(\bar{s} - s[t])) + (1 - \omega(\tau))(s[t] + \gamma(s[t] - \bar{s})) \\ &= s[t] + \gamma(s[t] - \bar{s}) + \omega(\tau)(\gamma + \delta)(\bar{s} - s[t]). \end{aligned} \quad (22)$$

⁷ Henceforth, it will not be emphasized that a macroeconomic variable is expressing the difference between, for example, the domestic and foreign price levels.

Then, combine the equations that describe the money and the international asset markets in equilibrium, i.e., (1)-(2), and substitute market expectations in (22) into the resulting equation:

$$\begin{aligned} m[t] - p[t] &= \bar{y} - \alpha(s[t] + \gamma(s[t] - \bar{s}) + \omega(\tau)(\gamma + \delta)(\bar{s} - s[t]) - s[t]) \\ &= \bar{y} - \alpha(\gamma(s[t] - \bar{s}) + \omega(\tau)(\gamma + \delta)(\bar{s} - s[t])). \end{aligned} \quad (23)$$

Differentiating (23) with respect to $m[t]$, $s[t]$ and \bar{s} gives

$$dm[t] = -\alpha(\gamma(ds[t] - d\bar{s}) + \omega(\tau)(\gamma + \delta)(d\bar{s} - ds[t])), \quad (24)$$

or, if (21) is substituted into (24),

$$\frac{ds[t]}{dm[t]} = \frac{d\bar{s}}{dm[t]} + \frac{1}{\alpha(\omega(\tau)(\gamma + \delta) - \gamma)} = 1 + o(\tau). \quad (25)$$

The current price level is held constant when deriving (25) since it is assumed to be sticky. Thus, (25) is the short-run effect on the exchange rate, near long-run equilibrium, of a change in money supply. A sticky price level also means that the quantity theory of money, i.e., (17), does not hold in the short-run since the price level is not affected by a monetary disturbance. Moreover, purchasing power parity, i.e., (20), does not either hold in the short-run since the exchange rate is affected by a monetary disturbance while the price level is not.

In order to have exchange rate overshooting, it must be true that

$$o(\tau) = \frac{1}{\alpha(\omega(\tau)(\gamma + \delta) - \gamma)} > 0, \quad (26)$$

which means that the planning horizon must satisfy

$$\tau > \log\left(1 + \frac{\gamma}{\delta}\right), \quad (27)$$

where the weight function in (5) is utilized in the derivation. Thus, in the short-run, before goods prices have time to react, the exchange rate will rise (fall) more than money supply, and, consequently, more than is necessary to bring the exchange rate to long-run equilibrium. It will turn out in the next two sections that (27) is also the stability condition for the model when it is assumed that market expectations are characterized by perfect foresight.

By letting $\tau \rightarrow \infty$ in (25), an equation describing exchange rate overshooting that corresponds to Dornbusch (1976) is obtained:

$$\left. \frac{ds[t]}{dm[t]} \right|_{Dornbusch(1976)} = 1 + \frac{1}{\alpha\delta}. \quad (28)$$

(28) corresponds to Dornbusch (1976) since, by letting $\tau \rightarrow \infty$ in (4)-(5), market expectations coincide with the expectations formed by fundamental analysis. In this case, the magnitude of exchange rate overshooting depends on the nominal interest rate response of real money demand (α), and the expected adjustment speed of the exchange rate according to fundamental analysis (δ).

Moreover, the magnitude of exchange rate overshooting depends inversely on the planning horizon:

$$\frac{do(\tau)}{d\tau} = \frac{do(\tau)}{d\omega(\tau)} \cdot \frac{d\omega(\tau)}{d\tau} = -\frac{\alpha(\gamma + \delta)}{\alpha^2(\omega(\tau)(\gamma + \delta) - \gamma)^2} \cdot \exp(-\tau) \leq 0, \quad (29)$$

i.e., for shorter planning horizons, more weight is placed on technical analysis, and since technical analysis is a destabilizing force⁸ in the foreign exchange market, the extent of exchange rate overshooting depends inversely on the planning horizon. This also means that the magnitude of overshooting is even larger in this model than in the Dornbusch (1976) model:

$$\frac{ds[t]}{dm[t]} \geq \frac{ds[t]}{dm[t]} \Big|_{Dornbusch(1976)}, \quad (30)$$

i.e., the exchange rate “overshoots the overshooting equilibrium”.

Finally, the magnitude of exchange rate overshooting depends on the structural parameters α , β , γ and δ in the following way⁹:

$$\frac{do(\tau)}{d\alpha} \Big|_{\tau \text{ given}} = -\frac{\omega(\tau)(\gamma + \delta) - \gamma}{\alpha^2(\omega(\tau)(\gamma + \delta) - \gamma)^2} < 0, \quad (31)$$

if exchange rate overshooting is assumed, i.e., if it is assumed that (27) holds,

$$\frac{do(\tau)}{d\beta} \Big|_{\tau \text{ given}} = 0, \quad (32)$$

$$\frac{do(\tau)}{d\gamma} \Big|_{\tau \text{ given}} = -\frac{\alpha(\omega(\tau) - 1)}{\alpha^2(\omega(\tau)(\gamma + \delta) - \gamma)^2} \geq 0, \quad (33)$$

and

$$\frac{do(\tau)}{d\delta} \Big|_{\tau \text{ given}} = -\frac{\alpha\omega(\tau)}{\alpha^2(\omega(\tau)(\gamma + \delta) - \gamma)^2} \leq 0. \quad (34)$$

Thus, the extent of exchange rate overshooting is larger, the less sensitive real money demand is to changes in the nominal interest rate (α), the faster the expected adjustment speed of the exchange rate is according to technical analysis (γ), and the slower the expected adjustment speed of the exchange rate is according to fundamental analysis (δ). The magnitude of exchange rate overshooting is not affected by changes in the degree of stickiness of goods prices (β).

3.3 Adjustment path to long-run equilibrium

To see how the exchange rate and the price level adjust to long-run equilibrium after a monetary disturbance, it is necessary to incorporate the price adjustment mechanism in (3). In order to do this, start with defining the expected change of the exchange rate as

$$\Delta^e s[t] \equiv s^e[t+1] - s[t], \quad (35)$$

⁸ Technical analysis is a destabilizing force since chartists expect that the exchange rate will diverge from long-run equilibrium. To see this, substitute (10) into (6).

⁹ The planning horizon in currency trade (τ) is given below in (31)-(34), but is endogenously determined when market expectations are characterized by perfect foresight. See the corresponding equations (69)-(72) below.

and, then, substitute market expectations in (22) into the definition in (35):

$$\Delta^e s[t] = \gamma(s[t] - \bar{s}) + \omega(\tau)(\gamma + \delta)(\bar{s} - s[t]) = (\gamma - \omega(\tau)(\gamma + \delta))(s[t] - \bar{s}). \quad (36)$$

(36) means that the market expect that the exchange rate will adjust to long-run equilibrium, if

$$\gamma - \omega(\tau)(\gamma + \delta) < 0, \quad (37)$$

which reduces to (27), i.e., if it is expected that the exchange rate will adjust to long-run equilibrium after a change in money supply, the exchange rate will also overshoot its long-run equilibrium level in the short-run.

Then, combine the equations that describe the money and the international asset markets in equilibrium, i.e., (1)-(2), and use the definition in (35):

$$m[t] - p[t] = \bar{y} - \alpha \Delta^e s[t]. \quad (38)$$

Thereafter, substitute the long-run equilibrium price level in (15) into (38):

$$\Delta^e s[t] = \frac{p[t] - \bar{p}}{\alpha}, \quad (39)$$

or, if (36) is substituted into (39),

$$s[t] = \bar{s} + \frac{p[t] - \bar{p}}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))}. \quad (40)$$

By using the relationship between the exchange rate and the price level in (40), its long-run counterpart in (19), and the price adjustment mechanism in (3), we can derive an equation that describes the adjustment path for the price level to long-run equilibrium.

In order to do this, start with defining the change of the price level as

$$\Delta p[t] \equiv p[t+1] - p[t], \quad (41)$$

and, then, substitute the definition in (41) into the price adjustment mechanism in (3):

$$\Delta p[t] = \beta(s[t] - p[t]). \quad (42)$$

Thereafter, substitute the relationship between the exchange rate and the price level in (40), and its long-run counterpart in (19), into (42):

$$\Delta p[t] = \left(\frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))} - \beta \right) (p[t] - \bar{p}). \quad (43)$$

(43) means that the price level will adjust to long-run equilibrium, if

$$\frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))} - \beta < 0, \quad (44)$$

or

$$\begin{cases} \tau < \log \frac{\alpha(\gamma + \delta)}{1 + \alpha\delta} \\ \tau > \log \left(1 + \frac{\tau}{\delta} \right) \end{cases}, \quad (45)$$

where the weight function in (5) is utilized in the derivation. The second equation in (45) is the same as (27). Moreover, the first equation in (45) implies that the market does not expect that the price level (nor the exchange rate) will adjust to long-run equilibrium, because (37) is not satisfied, even if the price level (and the exchange rate) will do that¹⁰. Consequently, this case is ruled out when market expectations are characterized by perfect foresight.

The difference equation in (43) can be written as

$$p[t+1] - \left(1 + \frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))} - \beta\right) p[t] = - \left(\frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))} - \beta\right) \bar{p}, \quad (46)$$

if the definition in (41) is utilized. Then, the solution of the difference equation in (46) is

$$p[t] = \bar{p} + \left(1 + \frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))} - \beta\right)^t (p[0] - \bar{p}), \quad (47)$$

and, after twice substituting the relationship between the exchange rate and the price level in (40)¹¹ into (47), the exchange rate's adjustment path is derived:

$$s[t] = \bar{s} + \left(1 + \frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))} - \beta\right)^t (s[0] - \bar{s}). \quad (48)$$

Thus, the exchange rate and the price level will adjust to long-run equilibrium after a monetary disturbance, if

$$\left|1 + \frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))} - \beta\right| < 1. \quad (49)$$

Moreover, the adjustment process is oscillating, if

$$-1 < 1 - \frac{\beta}{\alpha(1 - \gamma - (\gamma + \delta) \exp(-\tau))} - \beta < 0, \quad (50)$$

and non-oscillating, if

$$0 < 1 - \frac{\beta}{\alpha(1 - \gamma - (\gamma + \delta) \exp(-\tau))} - \beta < 1, \quad (51)$$

where the weight function in (5) is utilized in the derivations. For example, since the expression within the inequalities in (50)-(51) is larger when the planning horizon is longer, the adjustment process of the exchange rate (and the price level) to long-run equilibrium after a monetary disturbance is more likely to be oscillating for shorter than for longer planning horizons. Also recall that the magnitude of exchange rate overshooting depends inversely on the planning horizon. Thus, which is an interesting characteristic of the model, a comparably short planning horizon in currency trade is consistent with a highly variable and oscillating exchange rate.

3.4 Perfect foresight path

It is important that market expectations about future exchange rate movements are not arbitrary, and, given the model, do not involve (persistent) prediction errors. This is to say that the market should have

¹⁰ There is no exchange rate overshooting after a monetary disturbance in this case. See (25) and note that $-1 < o(\tau) < 0$ since the first equation in (45) implies that $\alpha(\omega(\tau)(\gamma + \delta) - \gamma) < -1$.

¹¹ (40) at time t is substituted first into (47), and then is (40) at time 0 substituted. Note that since (40) holds at any point in time, it also holds at time 0.

perfect foresight, which means that the first equation in (45) can be ruled out since it implies that the market does not expect that the exchange rate will adjust to long-run equilibrium, even if the exchange rate will do that. Thus, since the second equation in (45) is the same as (27), (27) is both the stability condition for the model as well as the condition for exchange rate overshooting.

In order to characterize the perfect foresight adjustment path to long-run equilibrium, start with defining the change of the exchange rate as

$$\Delta s[t] \equiv s[t+1] - s[t]. \quad (52)$$

Then, (48) evaluated one time period ahead is

$$s[t+1] = \bar{s} + \left(1 + \frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))} - \beta\right)^{t+1} (s[0] - \bar{s}), \quad (53)$$

which, together with (48), is substituted into the definition in (52):

$$\Delta s[t] = \left(\left(1 + \frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))} - \beta\right)^{t+1} - \left(1 + \frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))} - \beta\right)^t \right) (s[0] - \bar{s}). \quad (54)$$

Finally, using (48) in (54) gives the difference equation

$$\Delta s[t] = \left(\frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))} - \beta \right) (s[t] - \bar{s}), \quad (55)$$

which, of course, is similar to the difference equation in (46).

Clearly, for the market to have perfect foresight, it must be true that

$$\Delta^e s[t] = \Delta s[t], \quad (56)$$

or, according to (36) and (55),

$$\gamma - \omega(\tau_{pf})(\gamma + \delta) = \frac{\beta}{\alpha(\gamma - \omega(\tau_{pf})(\gamma + \delta))} - \beta, \quad (57)$$

where the left-hand side of (57) is the expected adjustment speed of the exchange rate and the right-hand side of (57) is the actual adjustment speed of the exchange rate. Moreover, τ_{pf} is the perfect foresight planning horizon. The general solution of (57) is

$$\tau_{pf} = f(\alpha, \beta, \gamma, \delta). \quad (58)$$

Thus, the perfect foresight planning horizon is a function of the structural parameters in the model, and, therefore, endogenously determined within the model.

In order to derive how the perfect foresight planning horizon is affected by changes in the structural parameters α , β , γ and δ , define

$$x_0 \equiv \gamma - \omega(\tau_{pf})(\gamma + \delta) = (\gamma + \delta) \exp(-\tau_{pf}) - \delta, \quad (59)$$

where the weight function in (5) is utilized in the second step, which means that (57) can be written as

$$x_0 = \frac{\beta}{\alpha x_0(\tau_{pf})} - \beta, \quad (60)$$

which has the solution

$$x_0 = -\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}}. \quad (61)$$

Since (27) implies that $x_0 < 0$, we must have that

$$x_0 = -\frac{\beta}{2} - \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}}. \quad (62)$$

Then, the perfect foresight planning horizon can be solved for by combining (59) and (62):

$$\tau_{pf} = -\log \underbrace{\left(1 - \frac{\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}} + \gamma}{\gamma + \delta}\right)}_{\equiv x_1}, \quad (63)$$

where $0 < x_1 \leq 1$ since $\tau_{pf} \geq 0$, i.e.,

$$\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}} + \gamma < \gamma + \delta, \quad (64)$$

which is soon utilized. Consequently,

$$\frac{d\tau_{pf}}{d\alpha} = -\frac{1}{x_1} \cdot \frac{dx_1}{d\alpha} = -\frac{1}{x_1} \cdot \frac{\beta}{\alpha^2} \cdot \frac{1}{2\sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}}} \cdot \frac{1}{\gamma + \delta} < 0, \quad (65)$$

$$\frac{d\tau_{pf}}{d\beta} = -\frac{1}{x_1} \cdot \frac{dx_1}{d\beta} = \frac{1}{x_1} \cdot \left(\frac{1}{2} + \left(\frac{\beta}{2} + \frac{1}{\alpha} \right) \cdot \frac{1}{2\sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}}} \right) \cdot \frac{1}{\gamma + \delta} > 0, \quad (66)$$

$$\frac{d\tau_{pf}}{d\gamma} = -\frac{1}{x_1} \cdot \frac{dx_1}{d\gamma} = \frac{1}{x_1} \cdot \frac{\gamma + \delta - \left(\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}} + \gamma \right)}{(\gamma + \delta)^2} > 0, \quad (67)$$

where (64) is utilized in the last step, and

$$\frac{d\tau_{pf}}{d\delta} = -\frac{1}{x_1} \cdot \frac{dx_1}{d\delta} = -\frac{1}{x_1} \cdot \frac{\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}} + \gamma}{(\gamma + \delta)^2} < 0. \quad (68)$$

The interpretation of (65)-(68) is that the perfect foresight planning horizon is longer, the less sensitive real money demand is to changes in the nominal interest rate (α), the more flexible goods prices (β) are, the faster the expected adjustment speed of the exchange rate according to technical analysis (γ) is, and the slower the expected adjustment speed of the exchange rate according to fundamental analysis (δ) is.

Finally, the effect on the magnitude of exchange rate overshooting of a change in the structural parameters, given perfect foresight, can be derived:

$$\frac{do(\tau_{pf})}{d\alpha} = \underbrace{\frac{do(\tau_{pf})}{d\alpha} \Big|_{\tau_{pf} \text{ given}}}_{<0} + \underbrace{\frac{do(\tau_{pf})}{d\tau_{pf}} \cdot \frac{d\tau_{pf}}{d\alpha}}_{\geq 0}, \quad (69)$$

$$\frac{do(\tau_{pf})}{d\beta} = \underbrace{\frac{do(\tau_{pf})}{d\beta} \Big|_{\tau_{pf} \text{ given}}}_{=0} + \underbrace{\frac{do(\tau_{pf})}{d\tau_{pf}} \cdot \frac{d\tau_{pf}}{d\beta}}_{\leq 0} \leq 0, \quad (70)$$

$$\frac{do(\tau_{pf})}{d\gamma} = \underbrace{\frac{do(\tau_{pf})}{d\gamma} \Big|_{\tau_{pf} \text{ given}}}_{\geq 0} + \underbrace{\frac{do(\tau_{pf})}{d\tau_{pf}} \cdot \frac{d\tau_{pf}}{d\gamma}}_{\leq 0}, \quad (71)$$

and

$$\frac{do(\tau_{pf})}{d\delta} = \underbrace{\frac{do(\tau_{pf})}{d\delta} \Big|_{\tau_{pf} \text{ given}}}_{\leq 0} + \underbrace{\frac{do(\tau_{pf})}{d\tau_{pf}} \cdot \frac{d\tau_{pf}}{d\delta}}_{\geq 0}, \quad (72)$$

where (29), (31)-(34) and (65)-(68) are utilized¹² in the derivations. Thus, the extent of exchange rate overshooting, given perfect foresight, is smaller the more flexible goods prices (β) are. Concerning changes in the other structural parameters in the model, the effects on the magnitude of exchange rate overshooting are ambiguous.

It is important to distinguish between (31)-(34) and (69)-(72). The former set of equations, i.e., (31)-(34), describes the *direct effect* on exchange rate overshooting of a change in the structural parameters, while the latter set of equations, i.e., (69)-(72), describes the *total effect* on exchange rate overshooting of a change in the structural parameters. For example, the direct effect of an increase in the nominal interest rate response of real money demand (α) is to decrease the magnitude of exchange rate overshooting (see the first term on the right-hand side of (69)). However, the *indirect effect* of an increase in this semi-elasticity is to increase the extent of exchange rate overshooting (see the second term on the right-hand side of (69)), and this is because a shorter perfect foresight planning horizon (see (65)) increases the magnitude of exchange rate overshooting (see (29)). Consequently, the total effect of an increase in this semi-elasticity is ambiguous (see (69)).

3.5 Simulations of the model

The simplifying assumption that the economy has, for a long time, been in long-run equilibrium before a monetary disturbance occurs, is not made in this section, and this is because it is a bit restrictive. Thus, the long-period moving average in (7)-(8) is no longer necessarily equal to the long-run equilibrium exchange rate as in (10). Instead, it is a function of all past exchange rates, but since this complicates the formal analysis, a small simulation study is accomplished in order to illustrate the behavior of the model.

Basically, the equations that are used in the simulations below are the equations that describe the benchmark model, i.e., (1)-(3), the expectations formations, i.e., (4)-(9), and the exchange rate in long-run equilibrium, i.e., (15) and (19). All these equations are reduced to two equations¹³. The first equation

¹² Of course, (29) and (31)-(34) also hold for the perfect foresight planning horizon.

¹³ (73) below is derived in the Appendix, and (74) below is (3) slightly rewritten.

is¹⁴

$$s[t] = \frac{1 + \alpha\delta - \alpha\delta \exp(-\tau)}{\alpha(\delta - \delta \exp(-\tau) - \gamma \exp(-\tau - v))} \cdot (m[t] - \bar{y}) - \frac{1}{\alpha(\delta - \delta \exp(-\tau) - \gamma \exp(-\tau - v))} \cdot p[t] - \frac{\gamma(\exp(-\tau) - \exp(-\tau - v))}{\delta - \delta \exp(-\tau) - \gamma \exp(-\tau - v)} \cdot \sum_{k=1}^{\infty} \exp(-kv) s[t-k], \quad (73)$$

and the second equation is

$$p[t+1] = \beta s[t] + (1 - \beta) p[t]. \quad (74)$$

In the previous sections, money supply was exogenously given. This assumption is now replaced with the assumption that money supply follows a stochastic process:

$$\begin{cases} m[t] = m[t-1] - 1 & \text{with probability } \varepsilon \\ m[t] = m[t-1] & \text{with probability } 1 - 2\varepsilon \\ m[t] = m[t-1] + 1 & \text{with probability } \varepsilon \end{cases} \quad (75)$$

In contrast to previous sections, the market does not have perfect foresight here, and this is because of the difficulty in deriving the exchange rate's perfect foresight path.

Obviously, it is not possible to illustrate the behavior of the model from all possible aspects. Therefore, we restrict the illustrations to three cases; a change in the degree of stickiness of goods prices (β), a change in the planning horizon in currency trade (τ), and a change in the distribution of weights given to current and past exchange rates (v). Specifically, the degree of stickiness of goods prices is assumed to be $\beta = 0.001$ and $\beta = 0.5$, respectively, the planning horizon in currency trade is assumed to be $\tau = 2$ and $\tau = 100$, respectively, and the parameter that determines the distribution of weights given to current and past exchange rates is assumed to be $v = 0.001$ and $v = 100$, respectively. In all three cases, the same time path of money supply is used, where the probability of a change in money supply is $2\varepsilon = 0.2$. See Figure 1.

[Figure 1 about here.]

The values of all other structural parameters in the model do not change in the simulations, and the number of time periods is 100¹⁵. See Table 1 for the values of all structural parameters in the simulations, and see Figures 2-9 for the time paths of the exchange rate.

[Table 1 about here.]

[Figures 2-9 about here.]

Firstly, by visual inspection of Figures 1-9, the exchange rate's variability is larger than the variability of money supply. This behavior of the model is also consistent with the model's behavior in the previous

¹⁴ Even if (8) is not explicitly utilized in the derivation of (73), the weights given to current and past exchange rates are still constrained by (8).

¹⁵ When deriving the exchange rate's time path in the simulations, the infinity symbol (∞) in (73) is replaced with 100.

sections, where (10) did hold (assuming equality in the equation), since the conditions for exchange rate overshooting and stability were the same. Secondly, an increase in the flexibility of goods prices (β) has two effects on the exchange rate's time path. The first effect is a decrease in the exchange rate's variability, and the second effect is that the exchange rate's adjustment path is non-oscillating when goods prices are almost completely rigid and oscillating when they are less rigid (e.g., compare Figures 2 and 6). The first effect is rather intuitive since goods prices absorb more of the monetary disturbance and, therefore, the exchange rate need not adjust as much for the economy to reach long-run equilibrium. The second effect is consistent with the model's behavior in the previous sections (see the discussion in connection with (50)-(51)).

Thirdly, a longer planning horizon in currency trade (τ) decreases the exchange rate's variability (e.g., compare Figures 2 and 4), and this, of course, is because less weight is placed on technical analysis when forming market expectations. Fourthly, an increase in the parameter that determines the distribution of weights given to current and past exchange rates (v) decreases the exchange rate's variability (e.g., compare Figures 2-3), and this is because an increase in the parameter has the same effect, in principle, as a longer planning horizon in currency trade (compare τ and v in (73)).

The model's behavior when the remaining structural parameters (i.e., α , γ and δ) change is not discussed in this paper. Of course, changing these parameters may also generate interesting results.

4 Concluding discussion

It was demonstrated in this paper that the exchange rate “overshoots the overshooting equilibrium” when chartists were introduced into a sticky-price monetary model due originally to Dornbusch (1976). Chartists were introduced since questionnaire surveys reveal that currency trade to a large extent is based on technical trading, where moving averages is the most commonly used technique. Moreover, the surveys also reveal that the importance of technical trading depends inversely on the planning horizon in currency trade. By implementing these observations theoretically, and deriving the exchange rate's perfect foresight path near long-run equilibrium, it was also demonstrated that the shorter the planning horizon is, the larger the magnitude of exchange rate overshooting.

The effects on the exchange rate's time path of changes in the model's structural parameters were also derived. To give one example of the predictions of the model developed, consider a change in the degree of stickiness of goods prices (β). According to (70), there are two effects on the magnitude of exchange rate overshooting; a direct effect and an indirect effect. More flexible goods prices will increase the perfect foresight planning horizon, which in turn decreases the extent of exchange rate overshooting. This is the indirect effect of a change in the structural parameter, and since there is no direct effect, the total effect of an increased flexibility of goods prices is a decrease in the magnitude of exchange rate overshooting.

Thus, if we refer to the first paragraph in the introductory section of this paper, a first step has been taken in developing an economic theory of exchange rate movements by taking into account observed behavior of currency traders. Of course, the present paper is not the only paper that takes this step, but in our opinion there are too few papers that use a chartist-fundamentalist setup in a foreign exchange model. Moreover, the chartists' expectations in those papers are very simple with, for example, no consideration taken for the role of the planning horizon in currency trade. Therefore, more research within this area is needed, where the chartists' expectations are based on other technical trading techniques than moving averages, and the fundamentalists' expectations are based on other macroeconomic models than the model used in this paper.

Appendix

Firstly, combine the equations that describe the money and the international asset markets in equilibrium, i.e., (1)-(2), and solve for the exchange rate:

$$s[t] = s^e[t+1] + \frac{m[t] - p[t] - \bar{y}}{\alpha}. \quad (76)$$

Secondly, substitute the expectations formed by technical and fundamental analyses, i.e., (6) and (9), into market expectations in (4):

$$\begin{aligned} s^e[t+1] &= \omega(\tau)(s[t] + \delta(\bar{s} - s[t])) + (1 - \omega(\tau))(s[t] + \gamma(s[t] - MA[t])) \\ &= (1 - \omega(\tau)(\gamma + \delta) + \gamma)s[t] + \omega(\tau)\delta\bar{s} - (1 - \omega(\tau))\gamma MA[t], \end{aligned} \quad (77)$$

and, then, substitute the long-period moving average in (7) into (77):

$$\begin{aligned} s^e[t+1] &= (1 - \omega(\tau)(\gamma + \delta) + \gamma)s[t] + \omega(\tau)\delta\bar{s} - \\ &\quad (1 - \omega(\tau))\gamma(1 - \exp(-v)) \sum_{k=0}^{\infty} \exp(-kv) s[t-k] \\ &= (1 - \omega(\tau)(\gamma + \delta) + \gamma)s[t] + \omega(\tau)\delta\bar{s} - \\ &\quad (1 - \omega(\tau))\gamma(1 - \exp(-v))s[t] - \\ &\quad (1 - \omega(\tau))\gamma(1 - \exp(-v)) \sum_{k=1}^{\infty} \exp(-kv) s[t-k] \\ &= (1 - \omega(\tau)\delta + (1 - \omega(\tau))\gamma \exp(-v))s[t] + \omega(\tau)\delta\bar{s} - \\ &\quad (1 - \omega(\tau))\gamma(1 - \exp(-v)) \sum_{k=1}^{\infty} \exp(-kv) s[t-k]. \end{aligned} \quad (78)$$

Thirdly, substitute (78) into (76):

$$\begin{aligned} s[t] &= (1 - \omega(\tau)\delta + (1 - \omega(\tau))\gamma \exp(-v))s[t] - \\ &\quad (1 - \omega(\tau))\gamma(1 - \exp(-v)) \sum_{k=1}^{\infty} \exp(-kv) s[t-k] + \\ &\quad \omega(\tau)\delta\bar{s} + \frac{m[t] - p[t] - \bar{y}}{\alpha}, \end{aligned} \quad (79)$$

and, then, solve for the exchange rate:

$$s[t] = \frac{\alpha\omega(\tau)\delta\bar{s} + m[t] - p[t] - \bar{y}}{\alpha(\omega(\tau)\delta - (1 - \omega(\tau))\gamma\exp(-v))} - \frac{(1 - \omega(\tau))\gamma(1 - \exp(-v))}{\omega(\tau)\delta - (1 - \omega(\tau))\gamma\exp(-v)} \cdot \sum_{k=1}^{\infty} \exp(-kv) s[t - k]. \quad (80)$$

Fourthly, the exchange rate in long-run equilibrium, according to (15) and (19), is

$$\bar{s} = m[t] - \bar{y}, \quad (81)$$

which is substituted into (80):

$$s[t] = \frac{(1 + \alpha\omega(\tau)\delta)(m[t] - \bar{y}) - p[t]}{\alpha(\omega(\tau)\delta - (1 - \omega(\tau))\gamma\exp(-v))} - \frac{(1 - \omega(\tau))\gamma(1 - \exp(-v))}{\omega(\tau)\delta - (1 - \omega(\tau))\gamma\exp(-v)} \cdot \sum_{k=1}^{\infty} \exp(-kv) s[t - k]. \quad (82)$$

Finally, substitute the weight function in (5) into (82), and (73) in Section 3.5 is derived.

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Table 1: The values of the structural parameters in Figures 2-9

Figure	β	τ	v	α	γ	δ
2	0.001	2	0.001	1	0.5	0.5
3	0.001	2	100	1	0.5	0.5
4	0.001	100	0.001	1	0.5	0.5
5	0.001	100	100	1	0.5	0.5
6	0.5	2	0.001	1	0.5	0.5
7	0.5	2	100	1	0.5	0.5
8	0.5	100	0.001	1	0.5	0.5
9	0.5	100	100	1	0.5	0.5

Note: β is the stickiness of goods prices, τ is the planning horizon in currency trade, v is the parameter that determines the distribution of weights given to current and past exchange rates, α is the nominal interest rate response of real money demand, γ is the expected adjustment speed of the exchange rate according to technical analysis, and δ is the expected adjustment speed of the exchange rate according to fundamental analysis.

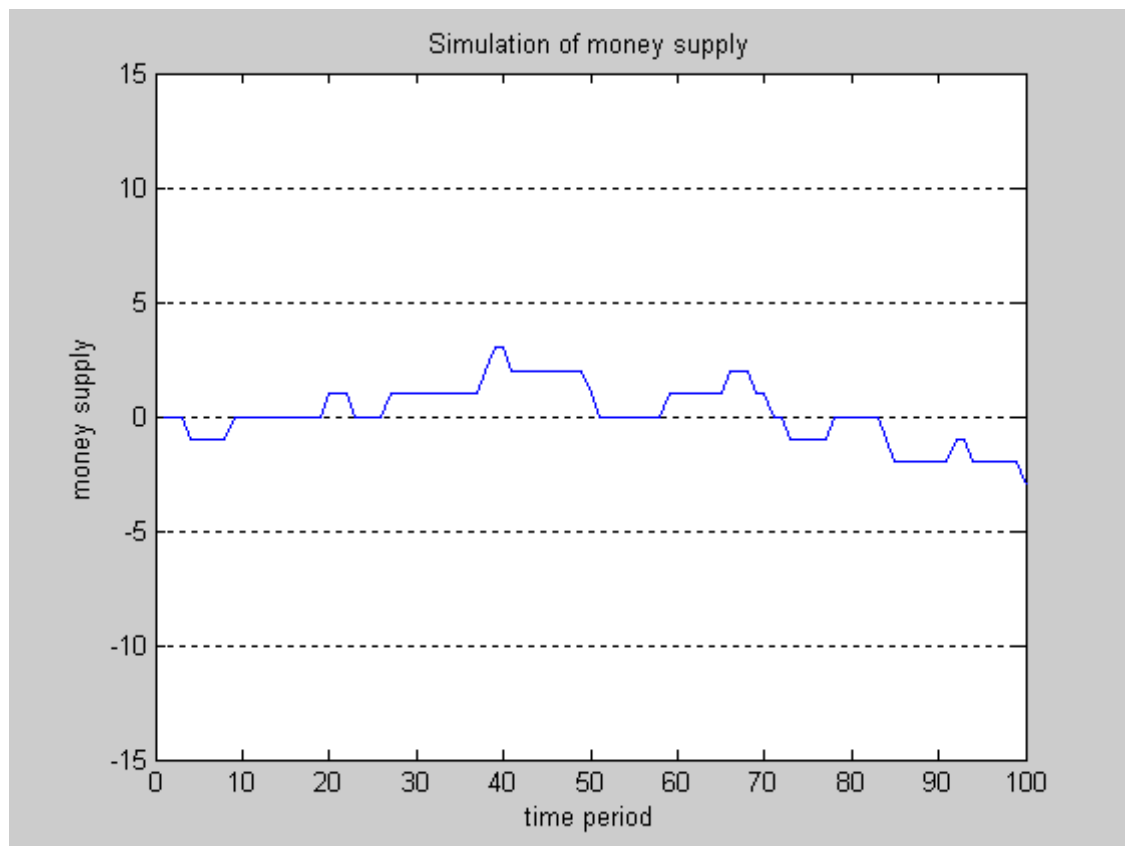


Figure 1:

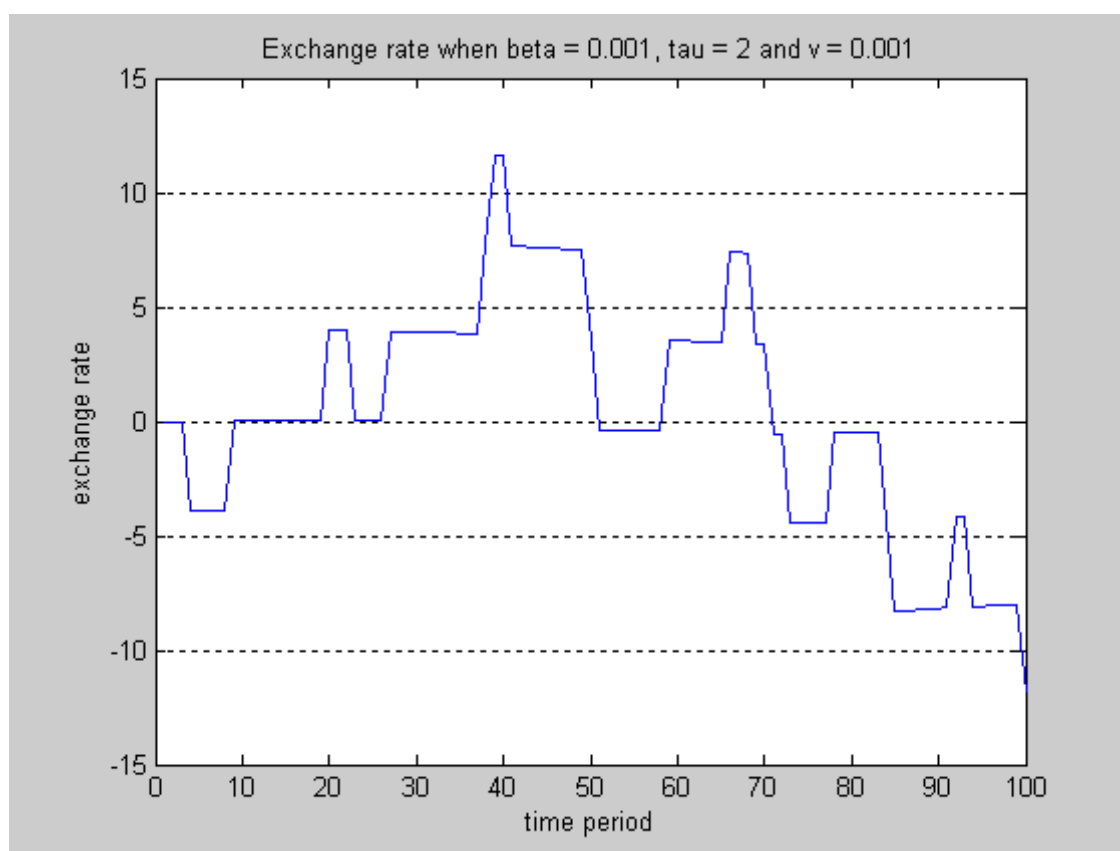


Figure 2:

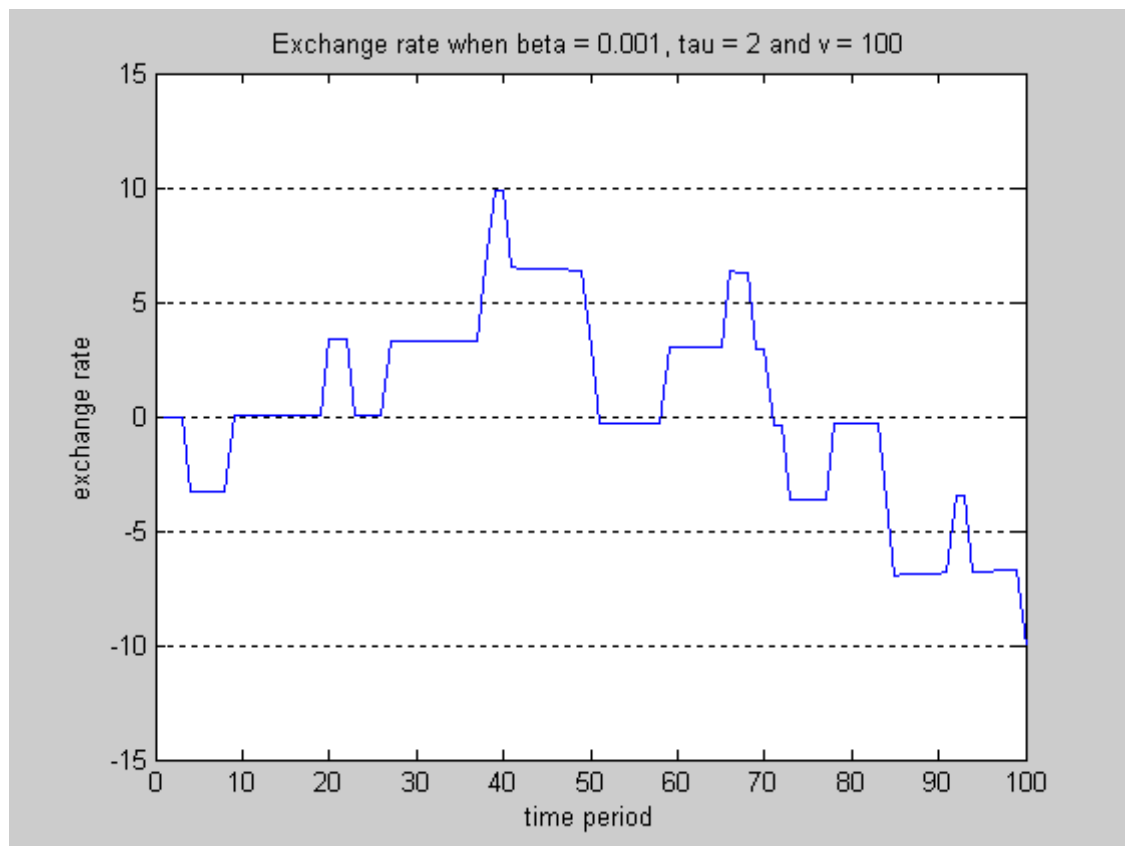


Figure 3:

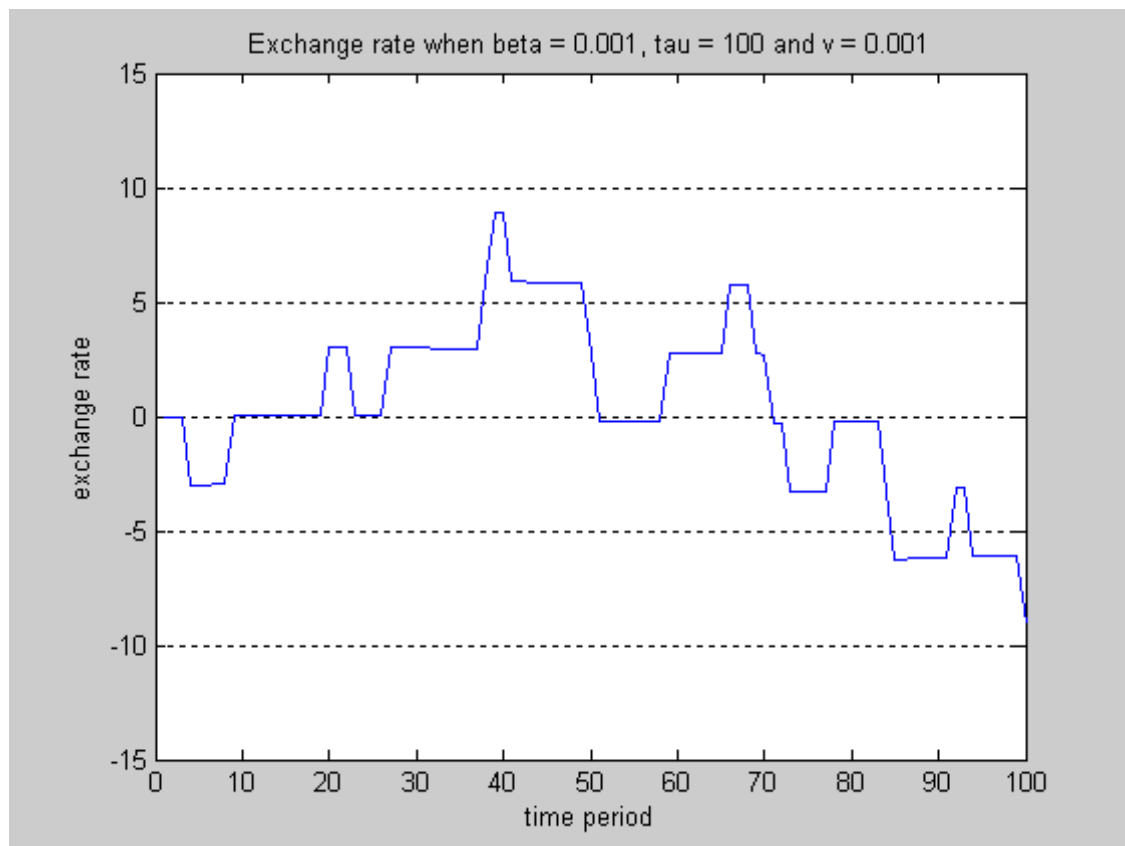


Figure 4:

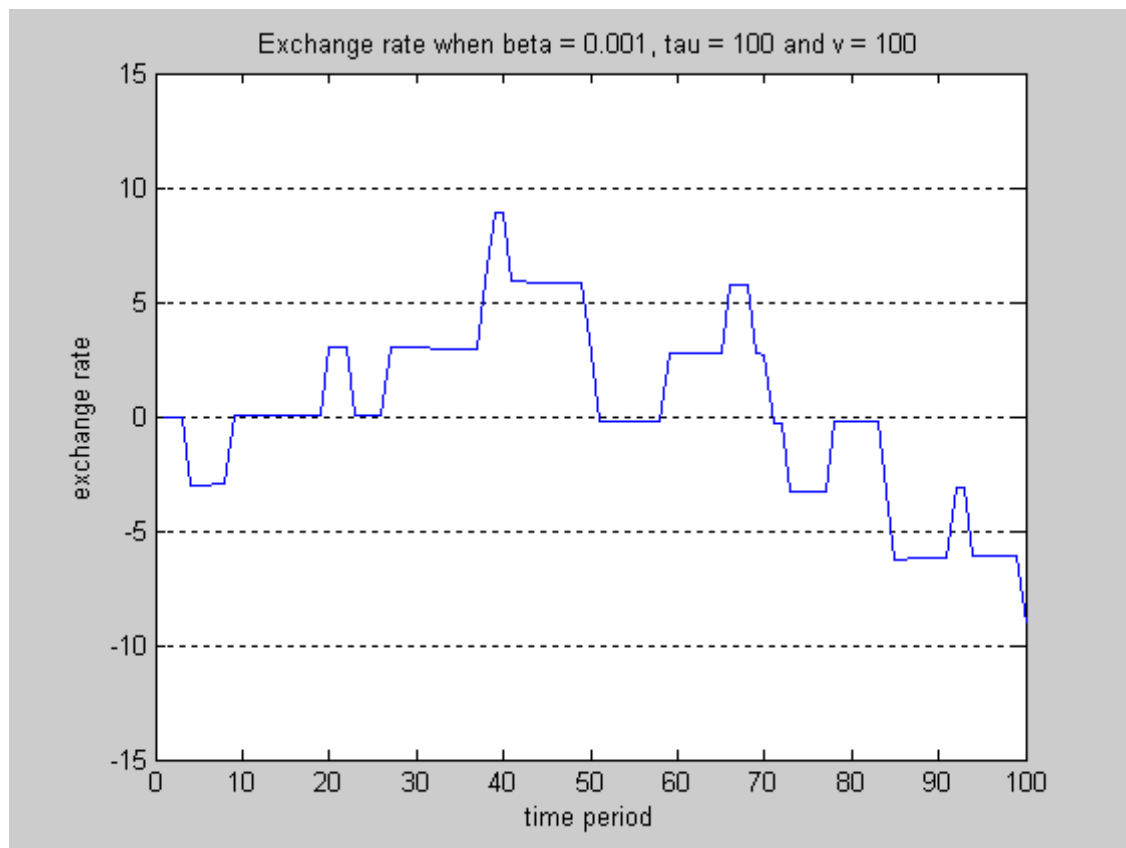


Figure 5:

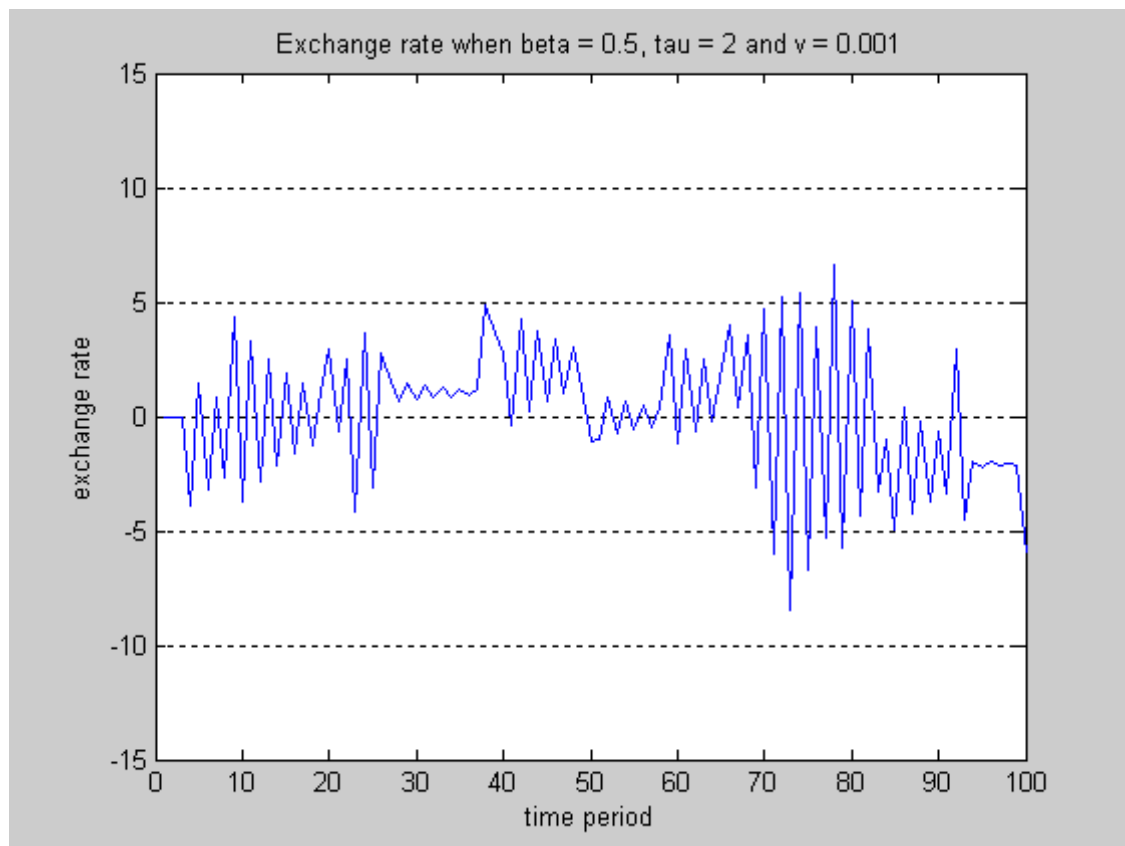


Figure 6:

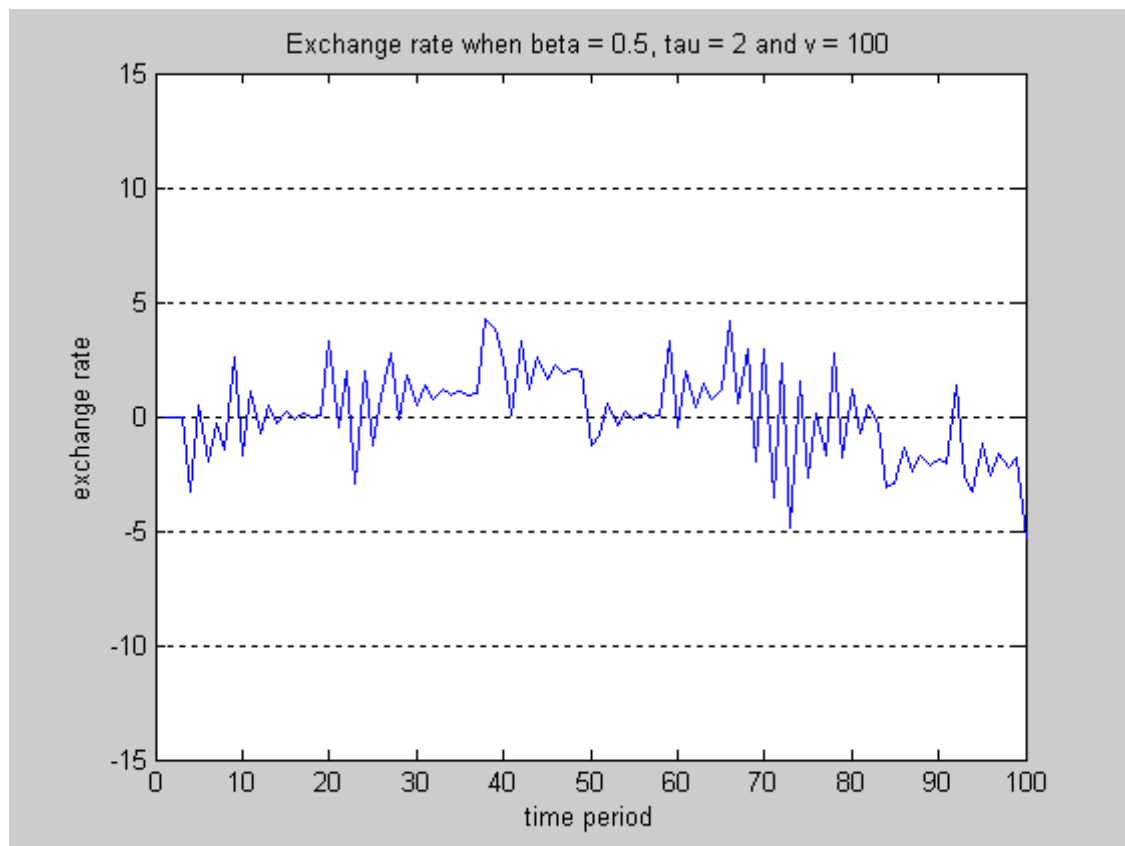


Figure 7:

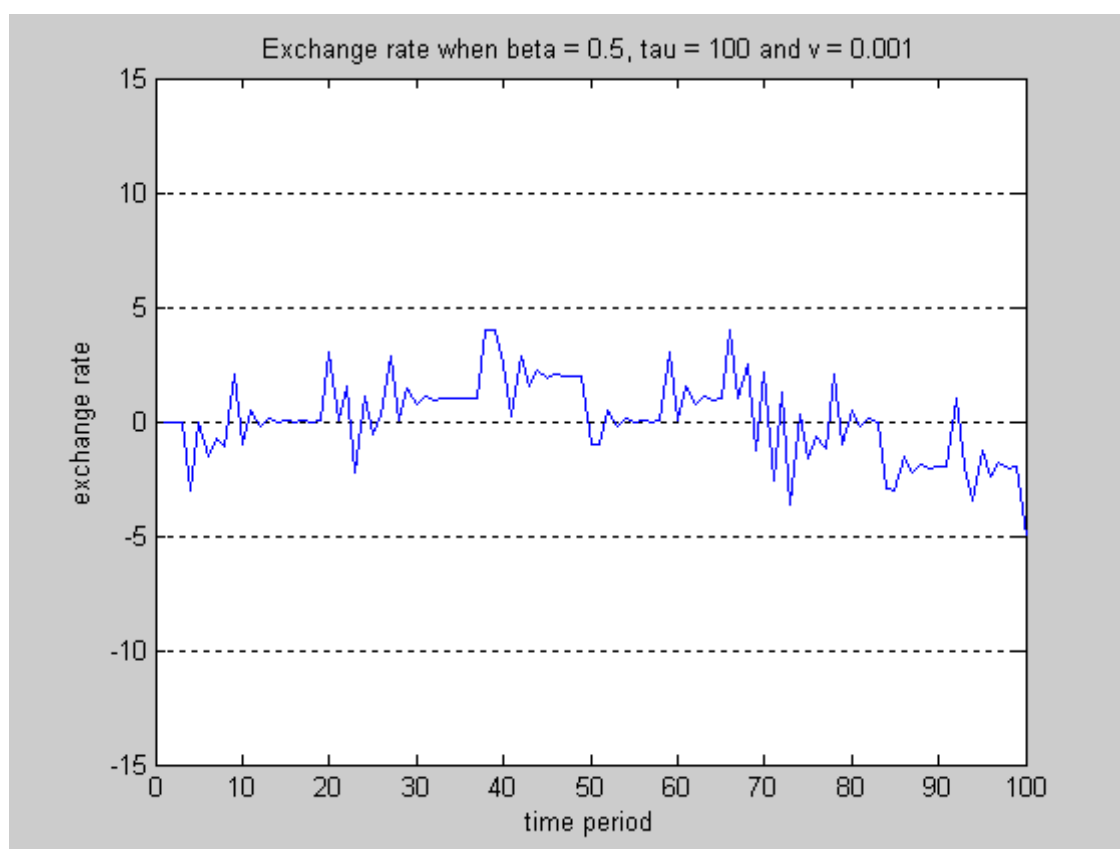


Figure 8:

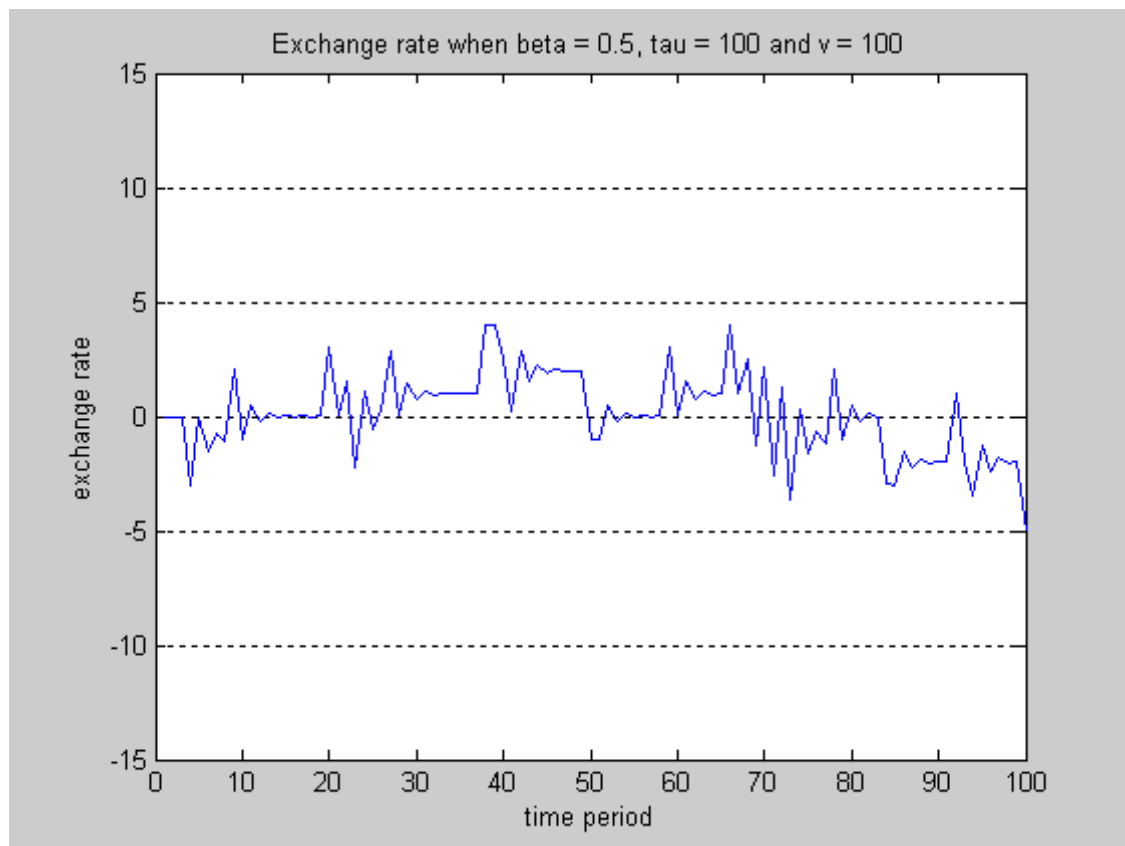


Figure 9: